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HYDRAULIC FUNDAMENTALS OF CLOSED
CONDUIT SPILLWAYS

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THIS PAPER

HYDRAULIC FUNDAMENTALS OF CLOSED CONDUIT SPILLWAYS

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Synopsis

Field and laboratory observations and tests have shown that a closed conduit spillway will flow completely full. This is true even if the barrel is on a slope steeper than the friction slope. It is not necessary that the outlet be submerged.

The capacity of the spillway can be computed and a head-discharge curve drawn using the ordinary equations for weir and pipe flow. The point at which the control changes from the weir to the pipe can also be readily determined. However, certain precautions that are not completely known at present must be considered to insure that two controls, such as orifice and pipe, do not exist in such a way as to give, at different times, different discharges at the same head over the crest.

Pressures within the spillway may be determined by means of laboratory models and projected to a prototype structure having a total fall, conduit length, or conduit roughness that is not similar to its model. It is only necessary that simple corrections be made for velocity, elevation, and friction.

Introduction

Closed conduit spillway is the name attached to the type of structure discussed here—spillway because it is used to convey excess water from one side of an embankment or dam to the other; closed conduit because its periphery is completely enclosed. The spillway consists of three parts—the inlet, the conduit, and the outlet. The inlet may take a number of forms. A pipe inlet having a number of entrance shapes and a drop inlet having a number of crest forms—including morning glory crests—are among the many forms the inlet may take. The closed conduit may be a prefabricated pipe of any material or a conduit of any cross-sectional form such as circular, rectangular, horseshoe, and so forth. Outlets range from simply letting the flow drop out the end of the conduit to elaborate stilling basins.

Most of this paper will be devoted to the hydraulic design of the closed conduit—the determination of the capacity of the spillway and the determination of the hydraulic pressures within the spillway. Inlets will be mentioned, but only briefly. The outlet portion of the closed conduit spillway will not be discussed.

The methods outlined here are for general application. Size of spillway is no limitation; the methods apply to the smallest highway culvert as well as to the largest morning glory spillway. The fall through the spillway can be small or great. Slope of the conduit is immaterial, at least so far as experiments now indicate. The range of slope tested in the laboratory is from 2-1/2% to 30%. In the cases considered the former slope is mild, the latter

steep. A closed conduit on a mild slope is taken as one whose barrel slope
is less than the slope of the friction grade line. As a result, the hydraulic
grade line is above the conduit and the conduit is flowing under pressure.
Conversely, when the barrel is on a steep slope the friction grade line falls
below the barrel and, if the friction grade line coincides with the hydraulic
grade line as it does over most of the conduit length, the pressures within the
conduit are less than atmospheric.

It is not necessary that the outlet be submerged for the conduit to flow full
when the conduit slope is steep. The outlet can discharge freely, yet the con-
duit may flow completely full. Some will say that this is impossible. Their
reasoning undoubtedly goes back to the fact that the greatest capacity of a
circular conduit on a steep slope is achieved when the depth of flow in the
conduit is 93% of the diameter. As A. P. Folwell,[2] M. ASCE, says, ".....the
maximum discharge occurs when the depth is 0.9 of the diameter. From
this it follows that a circular sewer can never flow full unless under a head."
This is not correct—at least as a general statement. A number of examples
may be cited as evidence.

Mr. C. J. Francis, Regional Engineer at Lincoln, Nebraska, reported by
letter dated August 3, 1942 to Mr. C. E. Ramser, M. ASCE, Soil Conservation
Service Research Specialist in Hydrology (now retired), that a closed conduit
spillway located near Crawford, Nebraska flowed full even though it was laid
on a steep slope. Photographic evidence enclosed by Mr. Francis is present-
ed in Fig. 1(b). The conduit is a 24-in. corrugated metal pipe laid on a 13%
slope. The entrance is shown in Fig. 1(a). As shown in Fig. 1(b), it is appar-
ent that the outlet is not submerged. The writer has computed the friction
slope to be 12%. This shows that the conduit is on a steep slope.

Further evidence is contained in a report by Richard P. Weeber.[3] Tests
were made in 1939 on two closed conduit spillways. One pipe is of steel, 103
ft long, 14 in. in diameter, and laid on an 18% slope. The writer has computed
the friction slope at the maximum discharge to be 9%. The conduit slope is
twice as steep as the friction slope. The other pipe is 8-in. vitrified tile, 120
ft long, and is laid on a 10% slope. The friction slope of this pipe is 12%.
This slope is mild at the maximum flow, but steep for most other flows. Both
of these pipes run completely full.

In addition to these field data, the Soil Conservation Service in cooperation
with the Minnesota Agricultural Experiment Station and the St. Anthony Falls
Hydraulic Laboratory has been testing closed conduit spillways intermittently
since 1940. Slopes as steep as 30% were used. If certain conditions are met,
these pipes flow completely full. The results of some of these tests have been
reported by the writer.[4]

The closed conduit spillway is of considerable importance. This is espe-
cially true when it is considered that the common highway culvert used by
highway engineers; the "trickle tubes", drop inlet culverts, and uncontrolled
pipe outlets used by the Soil Conservation Service for farm and ranch ponds,

2. "Sewerage," by A. P. Folwell, John Wiley & Sons, Inc., New York, N. Y.,
   10th Ed., 1929, p. 57.
3. "Results of Field Tests on Inclined Pipes Used in Earth Dam Construction,"
   by Richard P. Weeber, SCS, Ill-2, Edwardsville, Illinois, March 30, 1940,
   unpublished.
4. "Hydraulics of Drop-Inlet Pipe-Conduit Spillways," by Fred W. Blaisdell,
   thesis presented to the University of New Hampshire at Durham, April
   1950 as partial fulfillment of the requirements for the professional degree
   of Civil Engineer.

erosion and flood control dams; and the large shaft or morning glory spillways used on major dams; are all closed conduit spillways.

Mr. J. N. Bradley,[5] A.M. ASCE, has listed a world total of 22 morning glory spillways. The individual capacity of these spillways ranges from 1138 cfs for the Bassana Power Canal in Northern Italy to 61,000 cfs for the Holston and Watauga Dams in Tennessee. Their combined capacity is 512,808 cfs. The fall through these spillways ranges from 55 ft for the Silent Valley Dam in Ireland to 656 ft for the Sulak Dam in Russia, with an aggregate fall of 3468 ft. In contrast, the maximum capacity of any Soil Conservation Service closed conduit spillway is probably in the neighborhood of 2,500 cfs, while the controlled head may range from 10 ft to 40 ft. Sizes of conduit employed by the Soil Conservation Service may range from 8-in. diameter pipe to twin-barrel structures having individual barrels 6 ft square. These are small when compared to the morning glory spillways. However, the Soil Conservation Service has installed thousands of these closed conduit spillways. Arthur F. Moratz, Acting Chief of the Design and Construction Division, estimates conservatively that 850 of these structures were installed in 1952 alone in the eight states comprising the Upper Mississippi Region. In view of these figures, it seems probable that the total installed capacity and the total head controlled by all Soil Conservation Service structures exceed the totals given above for the 22 morning glory spillways which were known to Mr. Bradley in 1948. The major role played by the closed conduit spillway in the Soil Conservation Service program is quite apparent.

A dozen years ago, the Soil Conservation Service engineers—and the writer was one of them—considered that a pipe laid on a steep slope would not flow full. If a drop inlet was used, the maximum capacity was determined by assuming that the pipe entrance acted as an orifice. However, as a result of field experiences, some of which have been mentioned, the engineers began to doubt their design criteria. Therefore, the laboratory tests at the St. Anthony Falls Hydraulic Laboratory were initiated. As a result of these tests, design criteria have been set up for the type of drop inlet reported by the writer,[4] and now closed conduits are designed to flow full, no matter what their slopes may be. In other words, the conduit now may be laid close to the natural ground surface instead of excavating to a point where the conduit can be laid on a slope less than the friction slope, yet the full head through the structure, that is, from the headwater level to the conduit exit, is used when computing the maximum capacity. This permits the use of smaller conduits. Obviously, these factors reduce the cost of the structure. Additional benefits are anticipated as design criteria are established for other entrance types.

Until about 1949, the laboratory tests were directed toward verification of the model-prototype relationship and the perfection of methods of analyzing the test data. This part of the problem is now (1953) well in hand, and efforts can be devoted to a study of inlets. The tests to 1953 have shown that: (1) conduits on steep slopes will flow full; (2) the Froude model law can be used to scale laboratory tests to prototype sizes even though considerable air is sometimes mixed with the water; (3) the flow through the spillway can be computed using the hydraulic laws for weirs and pipes which are already known; and, (4) pressures within the spillway can be determined in a model and

5. "An Investigation Concerning the Prototype Behavior on Morning-Glory Spillways," by J. N. Bradley, Hydraulic Laboratory Report Hyd-254, Bureau of Reclamation, U. S. Department of the Interior, Denver, Colorado, November 19, 1948.

computed for its prototype even though the conduit length, roughness, or total fall is not reproduced exactly in the model.

Determination of Spillway Capacity

The manner in which the head-discharge curve is obtained and how to determine when the control changes from the crest of the spillway to the spillway acting as a closed conduit will be explained. A drop inlet closed conduit spillway is shown in Fig. 2. The following analysis is valid for conduit slopes either greater or less than the friction slope, although in Fig. 2 the conduit slope is greater than the friction slope. The method is also valid for other forms of the inlet. The head over the drop inlet crest is H. The conduit has a diameter D. The drop through the spillway is Z.

The flow through the spillway can be computed from the familiar weir and pipe equations. The head-discharge curve for weir control will be discussed first.

The head for use with the weir equation is measured above the crest of the spillway. The head-discharge curve for weir control at the spillway crest is computed using this head, the known length of the crest, and a coefficient of discharge. A weir curve conforming to the equation

$$Q = c \, L \, \sqrt{2g} \; H^{3/2} \tag{1}$$

is shown in Fig. 2. Here Q is the discharge, c a coefficient, L the crest length, and g the acceleration due to gravity. The fact that the spillway will eventually flow completely full is ignored for the moment; the weir is assumed to control the head-discharge relationship for purposes of computing this portion of the head-discharge curve.

The head-discharge relationship for closed conduit flow is computed next. The equation is

$$Q = A \sqrt{\dfrac{2g \, (H + Z)}{K_e + K_o + f\dfrac{\ell}{D} + f_r \dfrac{\ell_r}{4R_r} \left(\dfrac{A}{A_r}\right)^2}} \tag{2}$$

The computation is made by assuming that the spillway is completely full no matter how small the flow may be. The head for use in this equation is the head over the weir crest plus the drop through the spillway $(H + Z)$. Suitable loss coefficients for the entrance K_e, the outlet K_o, and pipe friction f must, of course, be assumed. The last term in the denominator under the radical is the friction loss in the drop inlet. The subscript r refers to the riser or drop inlet, A is the conduit area, ℓ the conduit length, and R_r the hydraulic radius of the drop inlet. The head-discharge curve for full conduit flow is plotted and shown in Fig. 2. Notice especially that the origin of heads for closed conduit flow has been shifted downward by the drop through the spillway.

Values of the coefficients c, K_e, and K_o for one type of closed conduit spillway may be found in the author's thesis.[4]

It is now possible to determine the point at which the control passes from the weir to the closed conduit. This point is at the intersection of the two

head-discharge curves. The weir at the spillway crest controls the head-discharge relationship up to the intersection of these two curves. At higher discharges, the closed conduit exercises control. The point at which the control changes from the weir to the closed conduit has been verified by experiment. However, the experiments have shown a slight rounding at the intersection of the two curves rather than the angular change obtained with the computed curves. This is the manner in which the head-discharge curve for the closed conduit spillway is determined. The solid lines give the true head-discharge relationships except for the rounding at the intersection of the curves; the dashed lines are imaginary only.

The head-discharge curve for a closed conduit spillway (the solid curve shown in Fig. 2) has characteristics that make it quite useful for flood control purposes. One advantage, so important that it cannot be overlooked, is: After the pipe equation controls the flow, most of the increase in flow into the reservoir is automatically stored for later release. Some marked reductions in peak discharges are possible. Reductions in peak outflow to 20% or less of the peak inflow are not uncommon in Soil Conservation Service practice. So far as is known to the writer, this principle has been used on only three large dams—the Kingsley Dam of the Central Public Power and Irrigation District in Nebraska, and the Heart Butte and Shade Hill Dams built by the Bureau of Reclamation in North Dakota. Mr. A. J. Peterka,[6] A.M. ASCE, described the performance of the Heart Butte spillway before the Summer Convention of the ASCE at Denver, Colorado in June 1952. The advantage mentioned above is sometimes a disadvantage because after pipe flow begins there is a very small increase in the capacity of the spillway, even though the head over the spillway increases rapidly. This is dangerous because a closed conduit spillway may have little reserve capacity for use in unanticipated emergencies. Another emergency spillway is, therefore, highly desirable.

Mr. M. M. Culp,[7] A.M. ASCE, has presented the design methods and the technical and economic advantages of considering detention or spillway storage, as he calls it. Mr. Culp says, "In almost every case where the effect of spillway storage has been included in the design, the saving in the cost of the dam has been several times as great as the added engineering costs involved."

The writer has shown how to compute the head-discharge curve for a closed conduit spillway that flows full; however, nothing has been said about what determines whether or not the conduit will flow full. This is largely determined by entrance conditions. To date, only one type of inlet has been thoroughly tested. This is a drop inlet 1-1/4 pipe diameters square in plan, 5 pipe diameters deep, and having a square-edged pipe entrance. This inlet is proportioned to insure that the conduit will flow completely full at the computed head over the crest of the spillway. The drop inlet may not flow full if it is less than 5 pipe diameters deep. The result is that the pipe entrance acts as an orifice to control the discharge. The head-discharge relationship for orifice control is shown as the dotted line in Fig. 2(b). As the head builds up along the orifice curve, the pipe will eventually fill and the discharge will jump from the orifice curve to the pipe curve. The head at which this occurs is fortuitous. It is necessary that the drop inlet shown in Fig. 2(a) be 5 pipe diameters deep to insure against the orifice exercising a control over the head-discharge

6. "Heart Butte Spillway and Outlet Works," by A. J. Peterka, Preprint of talk before Summer Convention of ASCE at Denver, Colorado, June 1952, U. S. Department of the Interior, Bureau of Reclamation, Denver (processed).
7. "The Effect of Spillway Storage on the Design of Upstream Reservoirs," by M. M. Culp, Agricultural Engineering, August 1948, p. 344.

relationship. If orifice control exists, pipe control can also exist and there will be two discharges at the same head so that the actual discharge at any instant is fortuitous. This is certainly an undesirable and dangerous condition that must be avoided by using designs that have been verified by laboratory tests.

It is apparent from the preceding discussion that the characteristics of the inlet must be known before one can be certain that a conduit laid on a steep slope will always fill at a definite point on the head-discharge curve. Very little is now (1953) known about those entrance characteristics that will insure full conduit flow. However, indications are that suppression of the contraction at the conduit entrance will permit a considerable reduction in the drop inlet height or even possible elimination of the drop inlet.

So far, no mention has been made as to how or why the conduit flows full. A description of this phenomenon is undoubtedly of interest. The tests made on transparent models have given a good picture of the flow conditions in the drop inlet spillway. At the lowest forms the barrel flows as an open conduit, that is, partly full, as shown in Fig. 3. There is, as can be seen, a considerable disturbance near the barrel entrance caused by the water falling down the drop inlet. At increasing flows, the disturbance occasionally seals off the barrel near its entrance, and a slug of water travels down the barrel. This is illustrated by the sequence shown in Fig. 4. The dark places in the pipe represent air carried in the water. In Fig. 4(b), the slug leaving the barrel has the appearance of a traveling hydraulic jump. Air is sucked in through the drop inlet until the slug leaves the barrel. Also in Fig. 4(b), a new slug is forming near the barrel entrance. With further increases in the flow the slugs, or traveling hydraulic jumps, form with increased frequency and eventually several jumps may be in the barrel at one time. At still higher flows, the conduit is completely filled with a mixture of water and air, as can be seen in Fig. 5. Finally, the water flow becomes so great that the air flow stops, and the conduit is completely filled with water alone.

The most interesting and important point regarding this phenomenon is that the sealing off, which causes the conduit to flow full, occurs near the barrel entrance when the outlet is not submerged and the barrel is on a steep slope.

The weir controls the head-discharge relationship until the air flow stops. The fact that slugs may be in the barrel or that the conduit may be full of an air-water mixture does not change this fact. As long as air is carried through the spillway, the control is at the crest acting as a weir. In making this statement, it is assumed that vortices through which air may be sucked are not present over the inlet.

Determination of Pressures

There are times when a knowledge of the pressures within the spillway is desirable. It is entirely possible for the spillway to be proportioned in such a way that pressures close to absolute zero will be obtained. The attendant danger of cavitation makes it necessary to determine whether pressures within the spillway will be so low that cavitation damage is probable.

When the conduit is flowing as an open channel, the pressures are close to atmospheric above the water surface and are almost equal to the water depth on the conduit invert. This is true except near the entrance, where impact of the water falling down the drop inlet increases the pressure.

For intermediate flow conditions, when slugs (traveling hydraulic jumps) and air pass through the conduit, the pressures fluctuate between those

obtained for open-channel flow and those obtained just after the air flow stops and the conduit flows completely full of water alone. Therefore, it is not necessary to determine the pressures for intermediate flow conditions if only extremes of pressure are desired.

Hunter Rouse,[8] M. ASCE, in his "Elementary Fluid Mechanics," discusses the variation of piezometric head in closed conduit flow. Dr. Rouse's analysis has been expanded and, for full conduit flow, a method has been developed of presenting the pressures observed in models in such a way that they can be applied readily to conduits of any length, total drop, or roughness. In other words, the pressure effects due to conduit length, total drop, and roughness are separated from those due to local disturbances, such as the conduit entrance. This is accomplished through the use of the Bernoulli equation

$$\frac{V_o^2}{2g} + \frac{p_o}{w} + z_o = \frac{V^2}{2g} + \frac{p}{w} + z + h_f \tag{3}$$

where V is the velocity, p is the pressure, w is the unit weight of water, z is the elevation, and h_f is the friction head loss. The subscript o denotes a reference point, which is taken as the center line of the conduit at the plane of its exit. There p_o/w and z_o are zero. Rearranging and dividing both sides of the equation by the reference velocity head—the mean velocity in the conduit—

$$\frac{p/w + z + h_f}{V_o^2/2g} = 1 - \left(\frac{V}{V_o}\right)^2 = \frac{h_n}{V_o^2/2g} \tag{4}$$

The sum of the pressure head p/w, the elevation head z, and the friction head h_f has been designated h_n. The sum of these heads is the difference between the friction grade line and the hydraulic grade line, as can be seen from Fig. 6(a) which shows a typical closed conduit spillway on a steep slope together with the friction grade line and the hydraulic grade line. The sign of the friction head is negative since the length of the conduit is negative or upstream from the reference point at the outlet.

Eq. 4 shows that the sum of the pressure head, the elevation head, and the friction head divided by the velocity head is a function only of the ratio of the local velocity V to the reference velocity V_o. For flow in any dynamically similar structure, the ratio V/V_o is identical for any point within the structure. It is also interesting to note that V/V_o is also identical for different discharges through the spillway. Therefore, if the ratio of h_n to the velocity head is determined for any point within the spillway, this ratio will be the same for any other flow or for the corresponding point in any geometrically similar spillway. Use of this fact is made in scaling the observed model pressures up to their prototype values.

The meaning of h_n is of interest. It can be seen in Fig. 6(a) that h_n is the difference between the hydraulic grade line and the friction grade line. The term h_n is then the hydraulic grade line for a hypothetical horizontal, frictionless conduit, since corrections are made for both elevation head and friction head. The hydraulic grade line for this horizontal, frictionless conduit is

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8. "Elementary Mechanics of Fluids," by Hunter Rouse, John Wiley & Sons, Inc., New York, N. Y., 1946, p. 82.

coincident with the friction grade line, except where local disturbances cause deviations. The hydraulic grade line for the horizontal frictionless conduit is shown in Fig. 6(b). This grade line can be determined from model studies for any type of entrance or other local disturbance.

In order to compute the pressure head p/w for a prototype structure, values of the velocity head $V_o^2/2g$, the elevation of the point in question z and the friction head h_f are determined from the discharge and the structure dimensions, and $h_n/(V_o^2/2g)$ is read from a plot similar to Fig. 6(b). These values are inserted in the equation

$$\frac{p}{w} = \frac{V_o^2}{2g}\left(\frac{h_n}{V_o^2/2g}\right) - z - h_f \tag{5}$$

Solution of this equation gives the desired pressure head. If the computed pressure is close to or below the vapor pressure of the liquid, cavitation can be anticipated. Changes in the design to eliminate the danger of cavitation should obviously be made.

Conclusion

This paper has presented evidence to show that a closed conduit spillway may flow full even though the barrel slope is steep and the outlet is not submerged. The manner in which the head-discharge curve is computed is presented and the method for the determination of the point at which the control changes from the weir to the pipe is given. Finally, a method for the determination of the pressures within the conduit is presented which has wider application than methods presented previously.

Acknowledgements

Fig. 1--Closed conduit spillway located near Crawford, Nebraska

(a) Entrance

(b) Outlet flows completely full although it discharges
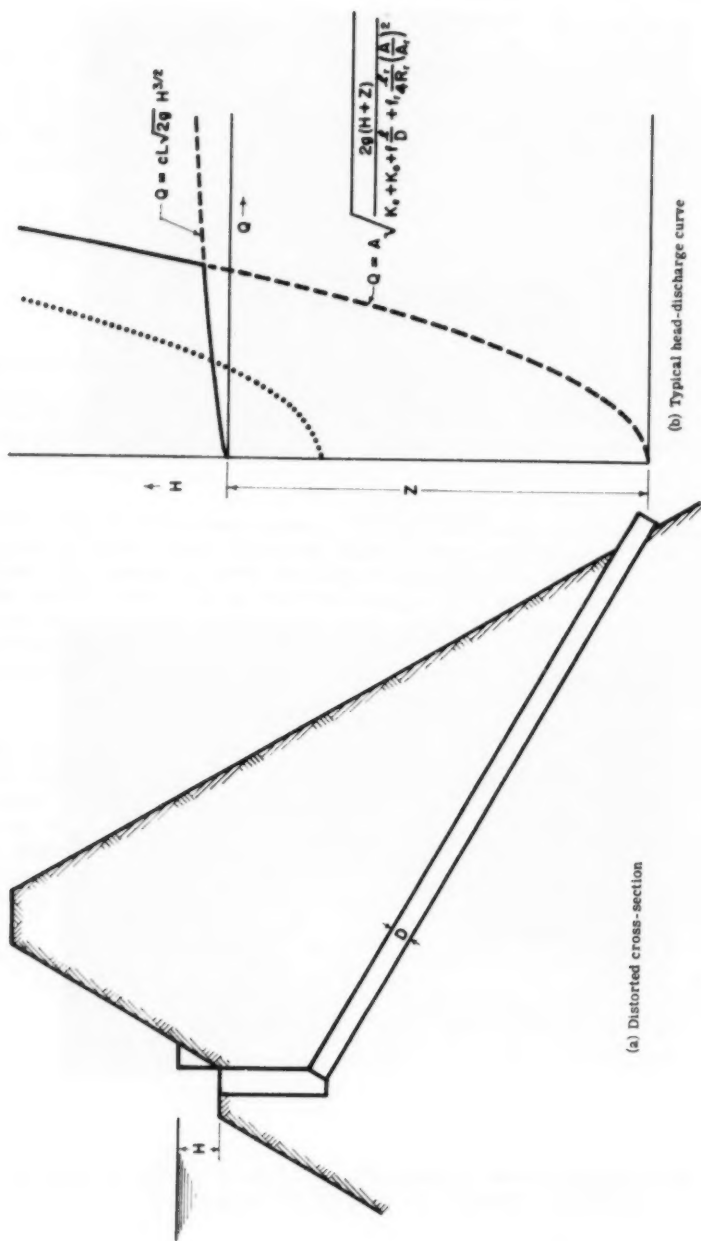freely. Barrel is on a steep slope.

$$Q = cL\sqrt{2g}\, H^{3/2}$$

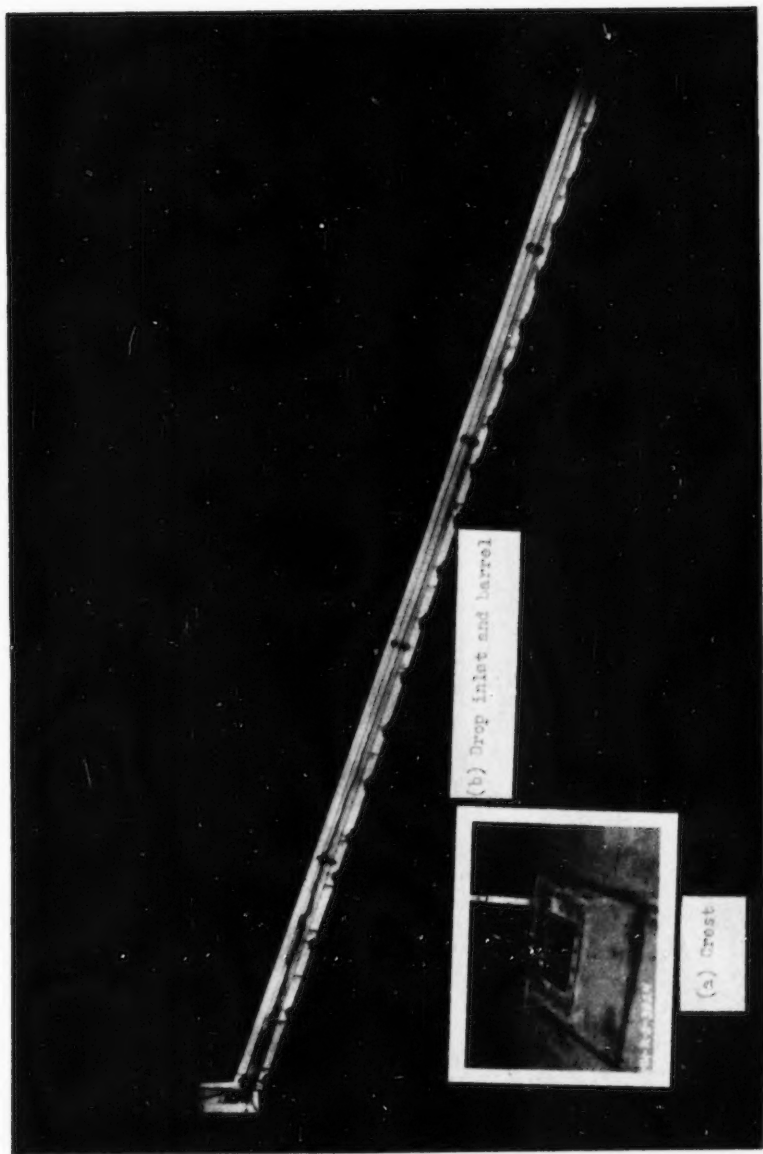$$Q = A\sqrt{\dfrac{2g(H+Z)}{K_e + K_b + 1 + f\dfrac{l}{D} + f_r\dfrac{Z_r}{4R_r}\left(\dfrac{A}{A_r}\right)^2}}$$

(a) Distorted cross-section

(b) Typical head-discharge curve

Fig. 2. Typical Closed Conduit Spillway

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(b) Drop inlet and barrel

(a) Crest

Fig. 3. Weir control at crest with barrel partly full

a. No slug in barrel. No air flow.

b. Slug leaving barrel. Air flow.

c. Barrel full of water-air mixture at upper end.
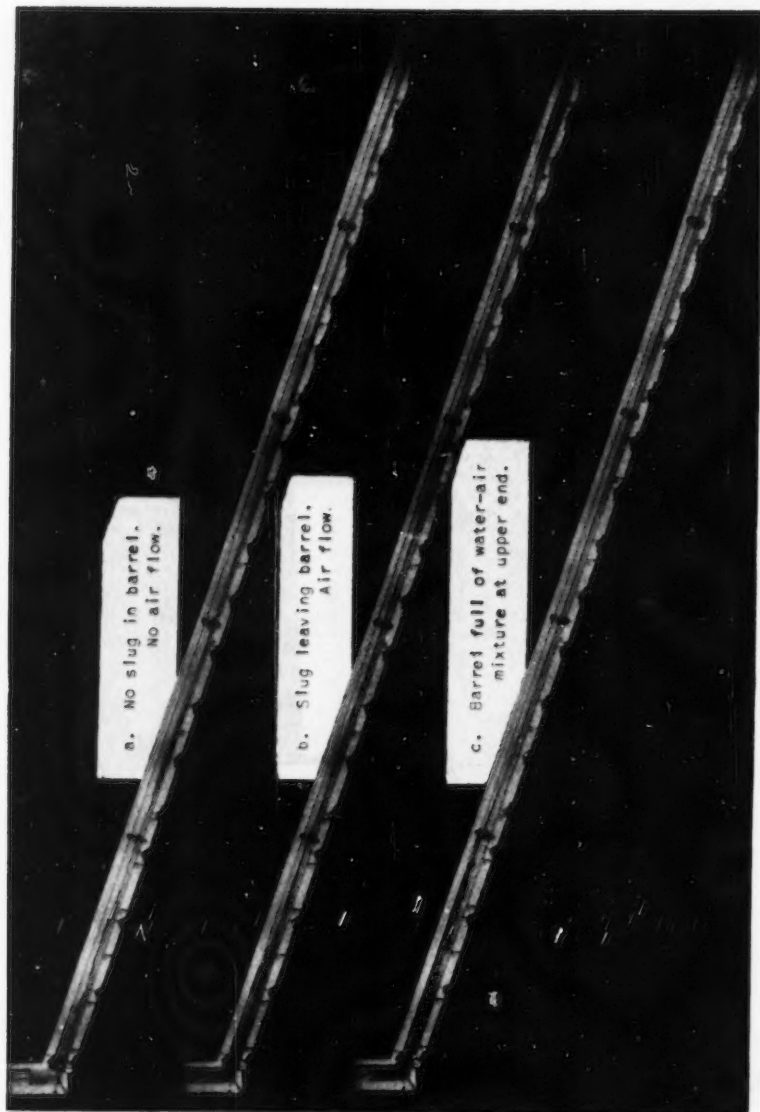
Fig. 4. Slug Flow Sequence
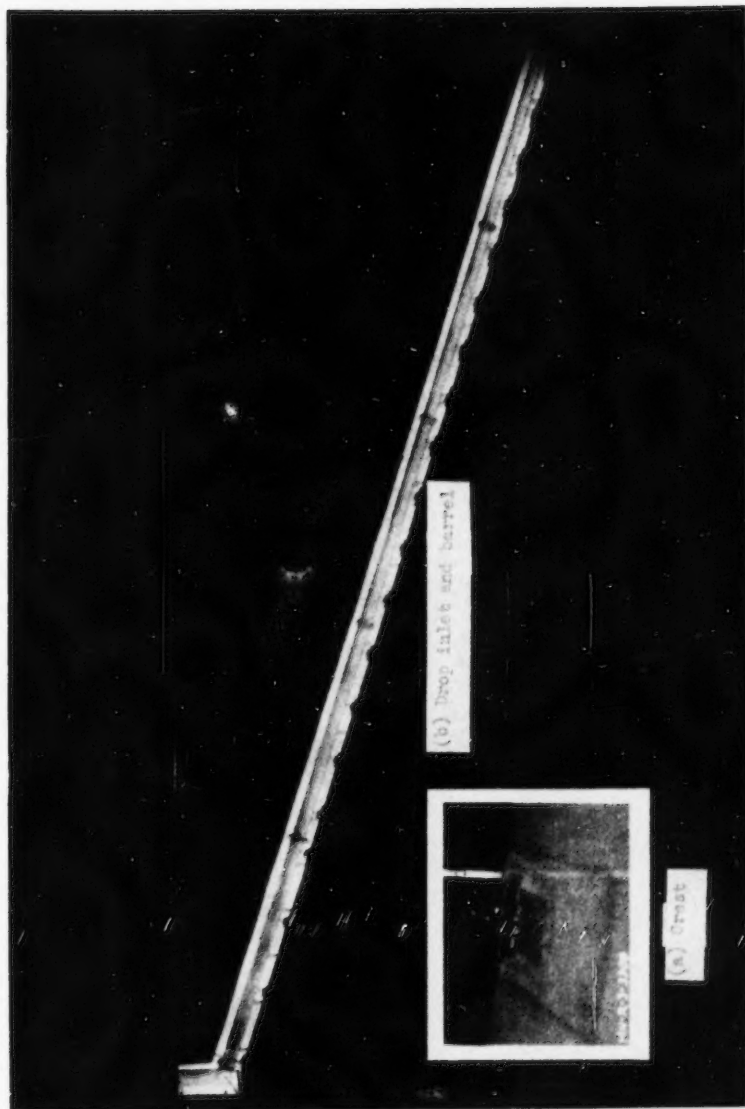
(b) Drop Inlet and Barrel

(a) Crest

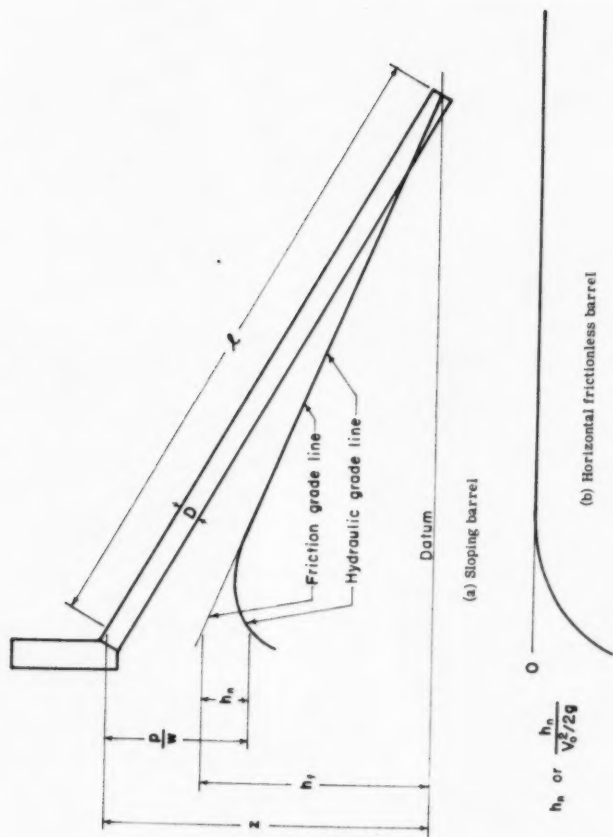Fig. 5. Barrel full of water-air mixture. Weir still controls head-discharge relationship.

Fig. 6. Typical Hydraulic Grade Line

(a) Sloping barrel

(b) Horizontal frictionless barrel